NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3811

CHARTS ADAPTED FROM VAN DRIEST'S TURBULENT FLAT-PLATE THEORY FOR DETERMINING VALUES OF TURBULENT AERODYNAMIC FRICTION AND HEAT-TRANSFER COEFFICIENTS

By Dorothy B. Lee and Maxime A. Faget

Langley Aeronautical Laboratory
Langley Field, Va.

Washington
October 1956

G

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3811

CHARTS ADAPTED FROM VAN DRIEST'S TURBULENT FLAT-PLATE THEORY FOR DETERMINING VALUES OF TURBULENT AERODYNAMIC FRICTION AND HEAT-TRANSFER COEFFICIENTS

By Dorothy B. Lee and Maxime A. Faget

SUMMARY

A modified method of Van Driest's flat-plate theory for turbulent boundary layer has been found to simplify the calculation of local skin-friction coefficients which, in turn, have made it possible to obtain through Reynolds analogy theoretical turbulent heat-transfer coefficients in the form of Stanton number. A general formula is given and charts are presented from which the modified method can be solved for Mach numbers 1.0 to 12.0, temperature ratios 0.2 to 6.0, and Reynolds numbers 0.2×10^6 to 200×10^6.

INTRODUCTION

Van Driest's theory has been found to be the most convenient of the turbulent theories as a means of estimating skin temperatures for the many flight trajectories at the Langley Pilotless Aircraft Research Division for various Mach numbers, temperature ratios, and Reynolds numbers; the theory has agreed well with experimental data. However, Van Driest's equation cannot be solved directly and a rapid means of obtaining theoretical values was desired. Inasmuch as the presentation of Van Driest's theory in the form of plots in sufficiently small increments for accurate and easy interpolation would require too many plots to be practical, a modification was devised which, with one equation and three charts, gives values of skin-friction coefficients in the range of Mach numbers 1.0 to 12.0, temperature ratios 0.2 to 6.0, and Reynolds numbers 0.2×10^6 to 200×10^6. This modified method along with the necessary charts is presented herein.

SYMBOLS

c_f local skin-friction coefficient, $\frac{2\tau_w}{\rho V^2} = \frac{\tau_w}{\frac{1}{2}\rho V^2}$

c_p specific heat of air at constant pressure, BTU/lb-°F

g	gravitational force, 32.2 ft/sec^2
h	aerodynamic heat-transfer coefficient, BTU/sec-ft^2-$^\circ$F
k	correction factor
M	Mach number
N_{St}	Stanton number, $\frac{h}{g\rho V c_p}$
R	Reynolds number, $\frac{\rho V x}{\mu}$
T	temperature, $^\circ$R
V	velocity, ft/sec
x	characteristic length, ft
Z	slope of straight-line approximation of c_f against R (fig. 1)
γ	ratio of specific heats, 1.4
ρ	density of air, slugs/cu ft
τ	shear stress, slugs/ft-sec^2 ?
μ	absolute viscosity of air, slugs/ft-sec
ω	constant in power law for viscosity, namely, $\mu_w = \mu_\infty\left(\frac{T_w}{T_\infty}\right)^\omega$

Subscripts:

w	wall
∞	free-stream condition
s	straight line

DISCUSSION

Method for Turbulent Flow

Van Driest's flat-plate theory for turbulent boundary layer (ref. 1) in the form assuming the Von Kármán similarity law for mixing length (ref. 2) is expressed in the equation for the local skin friction as

$$\frac{0.242}{A\left(c_{f,\infty}\right)^{1/2}\left(\frac{T_w}{T_\infty}\right)^{1/2}}\left\{\sin^{-1}\frac{A-\frac{B}{2A}}{\left[\left(\frac{B}{2A}\right)^2+1\right]^{1/2}}+\sin^{-1}\frac{\frac{B}{2A}}{\left[\left(\frac{B}{2A}\right)^2+1\right]^{1/2}}\right\} =$$

$$0.41 + \log_{10} R_\infty c_{f,\infty} - \omega \log_{10}\frac{T_w}{T_\infty} \qquad (1)$$

$\frac{1+\omega}{2} = 1.26 \rightarrow$ Linear mixing Length $\qquad \frac{\mu}{\mu_1} = \left(\frac{T}{T_1}\right)^{\omega}$

where

$$A^2 = \frac{\frac{\gamma-1}{2}M_\infty^2}{T_w/T_\infty} \qquad B = \frac{1+\frac{\gamma-1}{2}M_\infty^2}{T_w/T_\infty} - 1 \qquad \omega = 0.76$$

The value of $c_{f,\infty}$ obtained from equation (1) may be used to determine the heat-transfer coefficient in the form of Stanton number according to reference 3:

$$\boxed{N_{St} = 0.6c_f} \qquad 1.2 \qquad (2)$$

Some values of c_f were obtained from equation (1) for a range of $\frac{T_w}{T_\infty}$ from 0.2 to 6.0, Mach numbers from 1.0 to 12.0, and R from 0.2×10^6 to 200×10^6 with a punched-tape digital computer making as many as 11 iterations. The curves of c_f against R on log plots for given Mach numbers and temperature ratios were found to closely approximate a straight line faired through the values for Reynolds numbers of 1×10^6 and 40×10^6, as can be seen from the typical plots in figures 1(a) and 1(b) which are for Mach numbers of 1 and 5, respectively, and for $\frac{T_w}{T_\infty}$ of 0.2, 2.0, and 6.0. This straight line can be expressed

$$c_{f,s} = \frac{A}{R^Z} \tag{3}$$

and since $c_{f,s}$ is equal to c_f at $R = 1 \times 10^6$,

$$A = \left(c_f\right)_{R=1\times10^6} \left(1 \times 10^6\right)^Z$$

Thus,

$$c_{f,\cancel{s}} = \frac{\left(c_f\right)_{R=1\times10^6}}{\left(\frac{R}{1 \times 10^6}\right)^Z} \tag{4}$$

In order to evaluate Z it is necessary to substitute another set of values into equation (4), such as $R = 40 \times 10^6$ and c_f at $R = 40 \times 10^6$

$$\left(c_f\right)_{R=40\times10^6} = \frac{\left(c_f\right)_{R=1\times10^6}}{\left(\frac{40 \times 10^6}{1 \times 10^6}\right)^Z}$$

and

$$Z = \frac{\log \frac{\left(c_f\right)_{R=1\times10^6}}{\left(c_f\right)_{R=40\times10^6}}}{\log 40} \tag{5}$$

It can be seen that, with a chart of c_f at $R = 1 \times 10^6$ and a plot of Z as a function of M and $\frac{T_w}{T_\infty}$, values of $c_{f,s}$ can be obtained easily from equation (4) for any conditions of M, $\frac{T_w}{T_\infty}$, and R.

Values of Z for a range of Mach numbers from 1.0 to 12.0 at temperature ratios of 0.2 to 6.0 have been computed from equation (5) by using values of c_f at $R = 1 \times 10^6$ and $R = 40 \times 10^6$ obtained from the solution on the digital computer of Van Driest's equation. (See eq. (1).) The values of c_f are listed in table I. The values of Z are shown in figure 2(a) plotted against Mach number for several temperature ratios. A cross plot of Z against $\frac{T_w}{T_\infty}$ for several Mach numbers is shown in figure 2(b) which can conveniently be used to aid in interpolation. Accurate values of c_f calculated by the digital computer for a value of R of 1×10^6 for use in equation (4) are shown in figure 3(a) plotted against Mach number and are cross-plotted in 3(b) against $\frac{T_w}{T_\infty}$.

Because Van Driest's curves of c_f against R depart from the straight-line expression of equation (4), a correction factor k dependent on R is necessary for a higher degree of accuracy. The local skin-friction coefficient can then be written

$$c_f = kc_{f,s} \tag{6}$$

Values of k have been determined by comparing $c_{f,s}$ with solutions of Van Driest's equation for Mach numbers 1, 3, 6, 9, and 12, at temperature ratios of 0.2, 2.0, and 6.0, and at Reynolds numbers of 0.2×10^6, 4×10^6, 10×10^6, and 200×10^6. Figure 4 gives the correction factor k against R for the three temperature ratios and can be used for all Mach numbers from 1.0 to 12.0 since k proved to be almost invariant with Mach number.

The present method consists in solving equation (4) for $c_{f,s}$ by use of figures 2 and 3 and then determining c_f from equation (6) by use of figure 4. This procedure yields values of c_f well within ± 1.0 percent of Van Driest's theory. If $c_{f,s}$ is not corrected for k (fig. 4), the value obtained will be within ± 3.0 percent of the theory for R between 0.5×10^6 and 100×10^6.

Laminar Flow

As a matter of convenience a chart is included for the determination of N_{St} for laminar-flow conditions. Figure 5 is a plot of $N_{St}\sqrt{R_\infty}$ against M for various values of $\frac{T_w}{T_\infty}$ obtained from reference 4.

ACCURACY

The basic values of c_f at $R = 1 \times 10^6$ and $R = 40 \times 10^6$ (figs. 3(a) and 3(b) and table I), which were computed from Van Driest's equation (eq. 1) by the punched-tape digital computer with several iterations (approximately 11), are correct to four significant figures.

The curves of Z (figs. 2(a) and 2(b)) are faired smooth from point to point and when used in equation (4) will give essentially exact agreement at $R = 1 \times 10^6$ and $R = 40 \times 10^6$.

The maximum error in k due to neglecting the influence of M is ±0.75 percent.

CONCLUDING REMARKS

A modified method of Van Driest's flat-plate theory for turbulent boundary layer has been found to simplify the calculation of local skin-friction coefficients which, in turn, have made it possible to obtain through Reynolds analogy theoretical turbulent heat-transfer coefficients in the form of Stanton number. A general formula is given and charts are presented from which the modified method can be solved for Mach numbers 1.0 to 12.0, temperature ratios 0.2 to 6.0, and Reynolds numbers 0.2×10^6 to 200×10^6.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., June 20, 1956.

$(.72)^{2/3} = .831$ 1.204

REFERENCES

1. Van Driest, E. R.: Turbulent Boundary Layer in Compressible Fluids. Jour. Aero. Sci., vol. 18, no. 3, Mar. 1951, pp. 145-160, 216.

2. Van Driest, E. R.: The Turbulent Boundary Layer With Variable Prandtl Number. Rep. No. AL-1914, North American Aviation, Inc., Apr. 2, 1954.

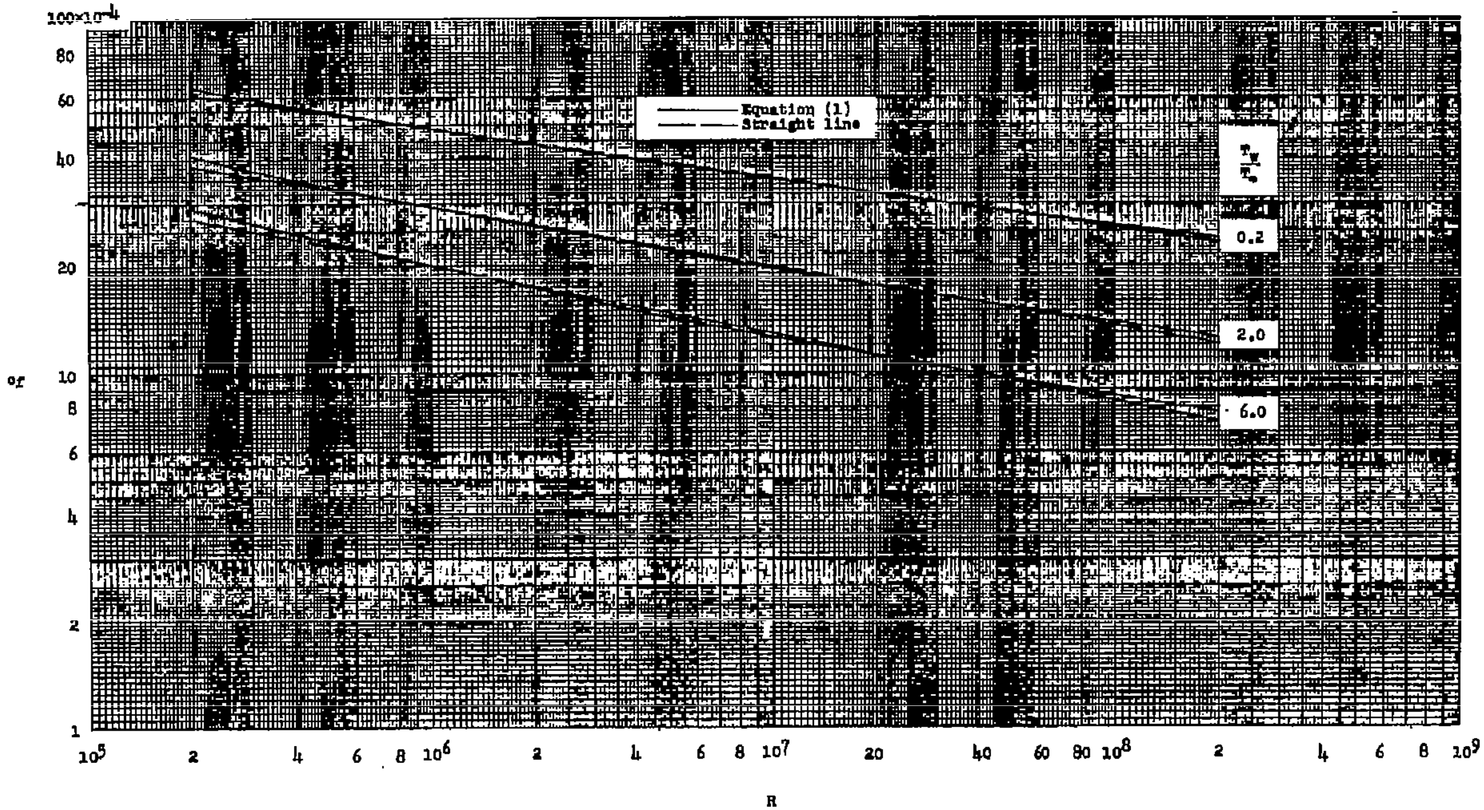
3. Rubesin, Morris W.: A Modified Reynolds Analogy for the Compressible Turbulent Boundary Layer on a Flat Plate. NACA TN 2917, 1953.

4. Van Driest, E. R.: Investigation of Laminar Boundary Layer in Compressible Fluids Using the Crocco Method. NACA TN 2597, 1952.

TABLE I

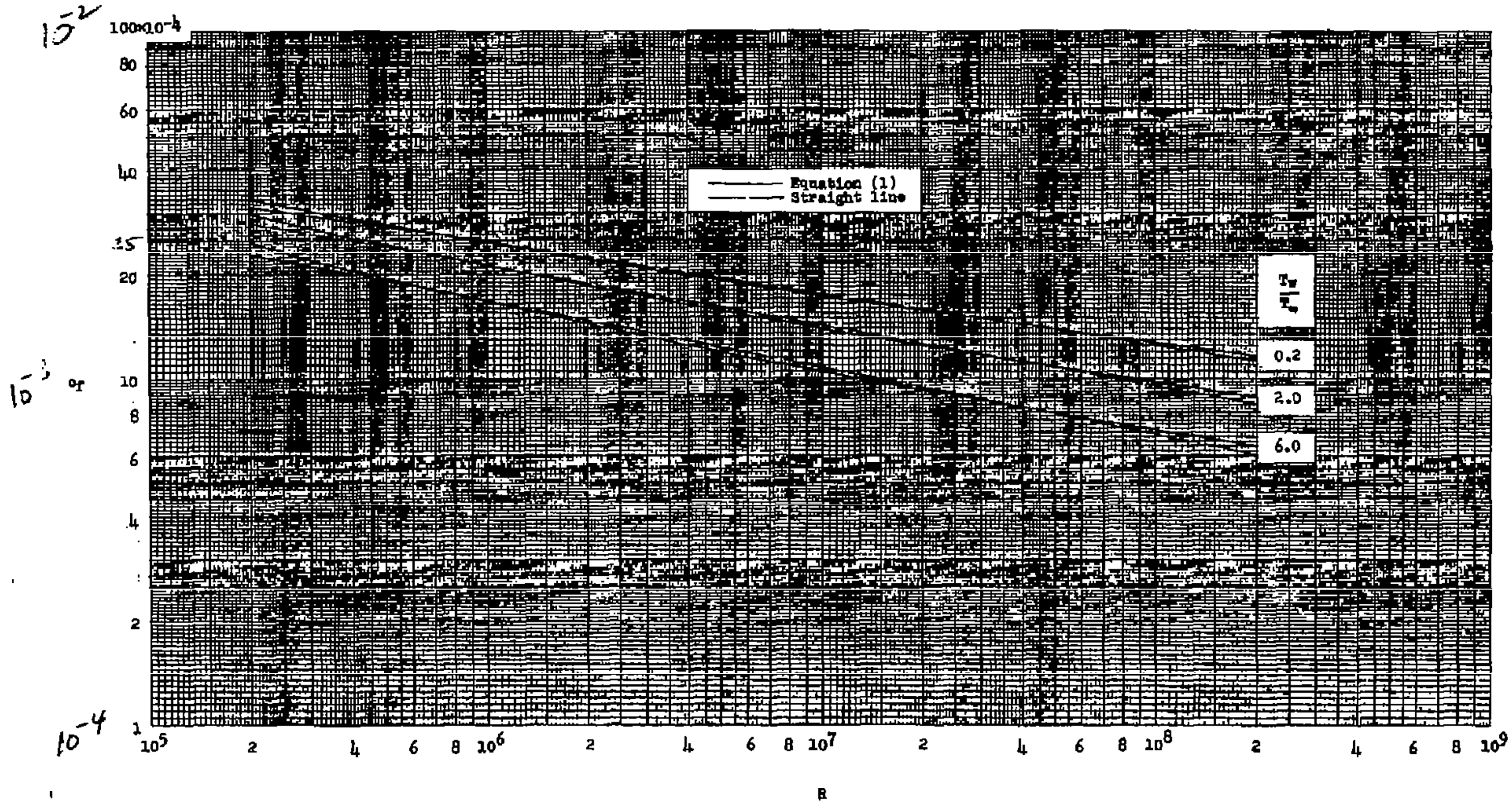
VALUES OF SKIN-FRICTION COEFFICIENT

M	$\frac{T_w}{T_\infty} = 0.2$	$\frac{T_w}{T_\infty} = 0.6$	$\frac{T_w}{T_\infty} = 1.0$	$\frac{T_w}{T_\infty} = 2.0$	$\frac{T_w}{T_\infty} = 3.0$	$\frac{T_w}{T_\infty} = 4.0$	$\frac{T_w}{T_\infty} = 6.0$
c_f at $R = 1 \times 10^6$							
0.5	0.005100	0.004170	0.003673	0.002990	0.002610	0.002350	0.002005
1.0	.004904	.004070	.003603	.002948	.002578	.002328	.002000
2.0	.004290	.003709	.003345	.002800	.002475	.002251	.001950
3.0	.003596	.003258	.003005	.002593	.002328	.002137	.001875
4.0	.002978	.002812	.002652	.002357	.002153	.002000	.001779
5.0	.002475	.002417	.002325	.002126	.001974	.001854	.001675
6.0	.002078	.002084	.002036	.001907	.001798	.001708	.001566
7.0	.001764	.001809	.001788	.001711	.001635	.001568	.001460
8.0	.001516	.001581	.001582	.001539	.001489	.001441	.001357
9.0	.001317	.001394	.001407	.001389	.001357	.001323	.001262
10.0	.001156	.001237	.001258	.001259	.001241	.001218	.001173
12.0	.000913	.000995	.001025	.001047	.001046	.001039	.001017
c_f at $R = 40 \times 10^6$							
.5	.003033	.002383	.002048	.001607	.001370	.001210	.001000
1.0	.002915	.002321	.002007	.001583	.001350	.001195	.000997
2.0	.002537	.002107	.001857	.001499	.001293	.001154	.000970
3.0	.002113	.001838	.001660	.001382	.001211	.001091	.000930
4.0	.001736	.001577	.001456	.001250	.001115	.001017	.000879
5.0	.001433	.001346	.001267	.001120	.001016	.000938	.000824
6.0	.001193	.001152	.001102	.000999	.000921	.000860	.000767
7.0	.001006	.000993	.000962	.000890	.000833	.000785	.000711
8.0	.000859	.000862	.000845	.000796	.000754	.000717	.000658
9.0	.000742	.000754	.000746	.000714	.000683	.000655	.000608
10.0	.000646	.000666	.000663	.000643	.000620	.000599	.000562
12.0	.000505	.000530	.000533	.000528	.000517	.000505	.000482



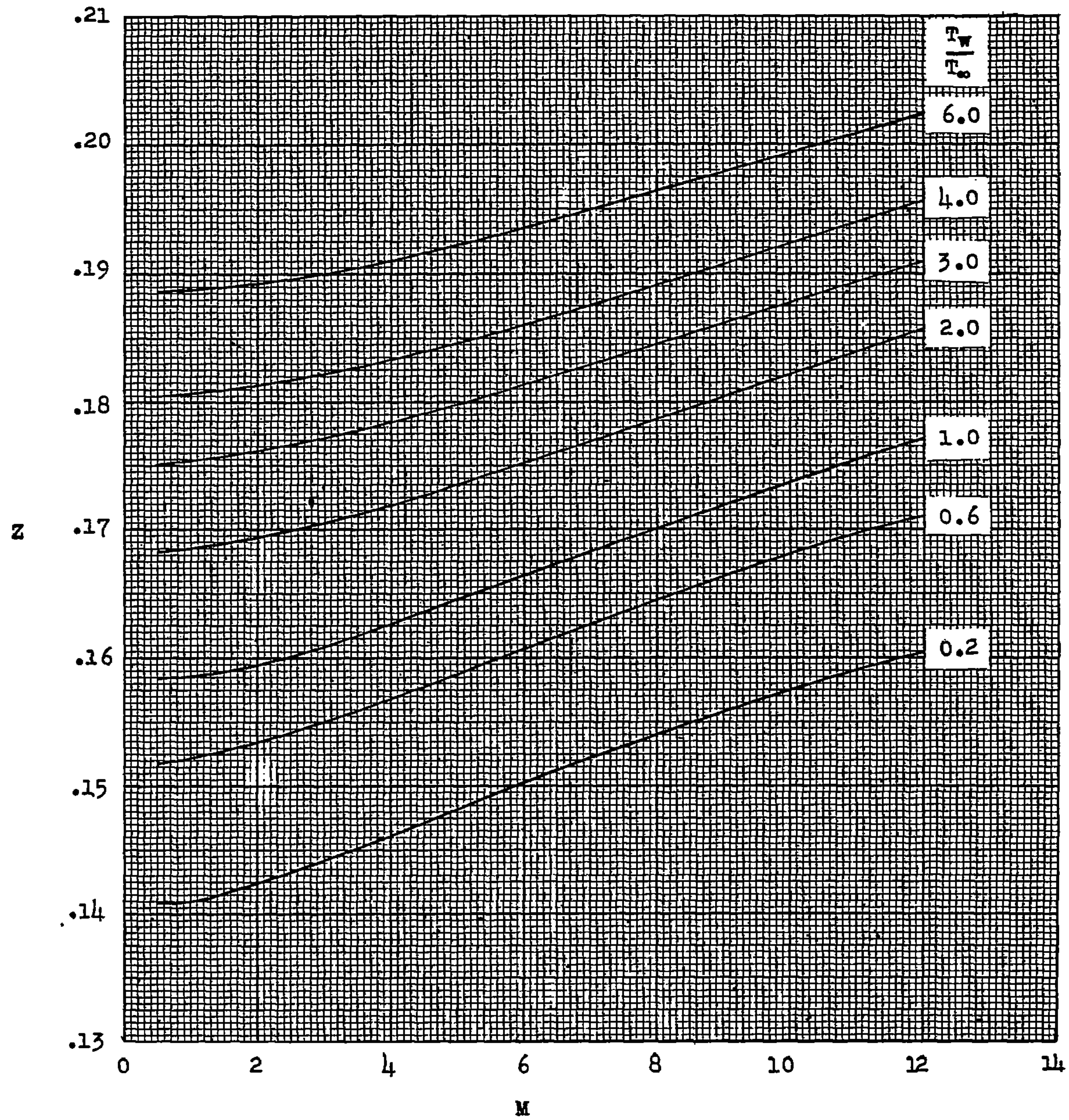
(a) Mach number of 1.

Figure 1.- Variation of local skin-friction coefficient with Reynolds number.



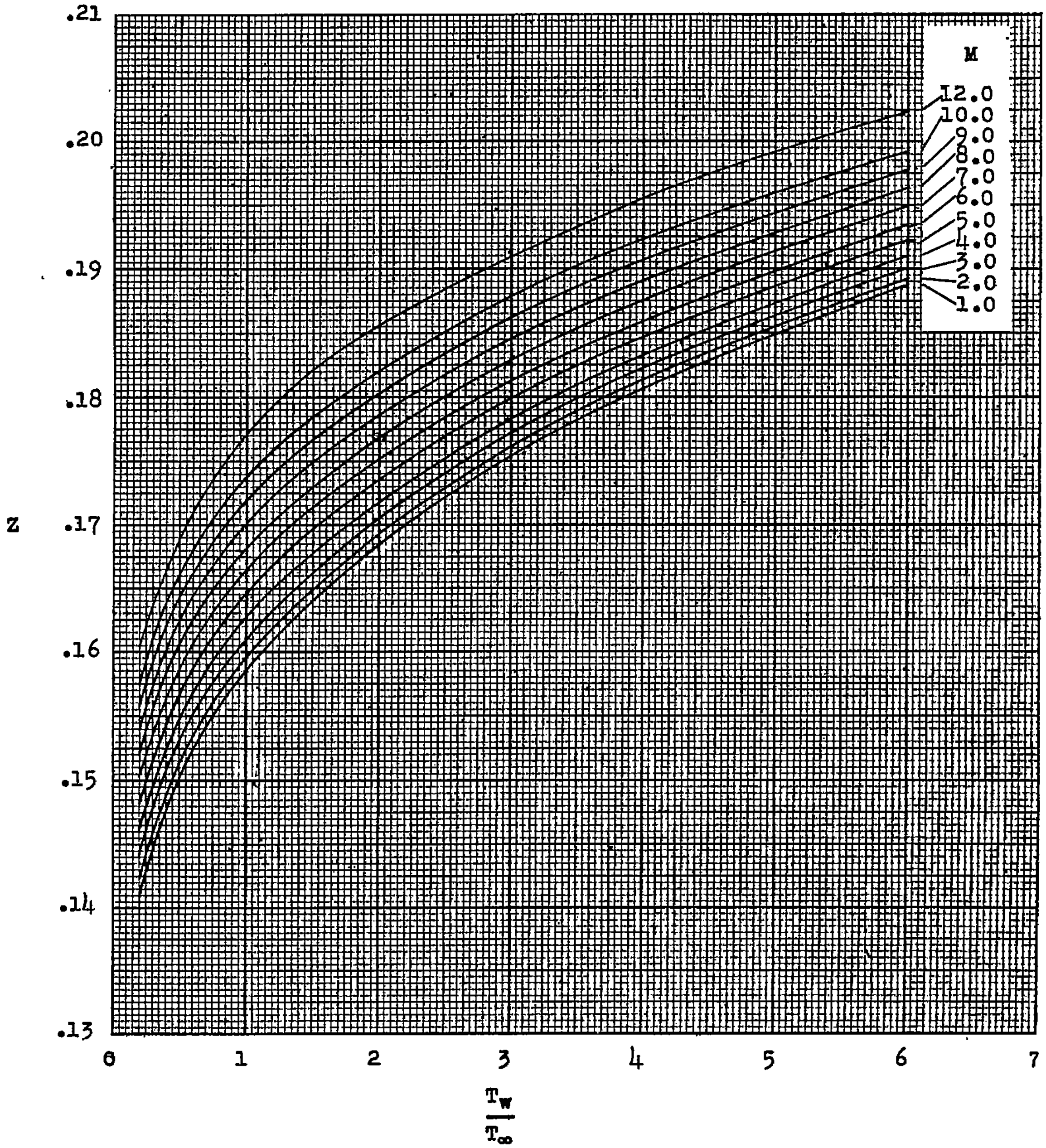
(b) Mach number of 5.

Figure 1.- Concluded.



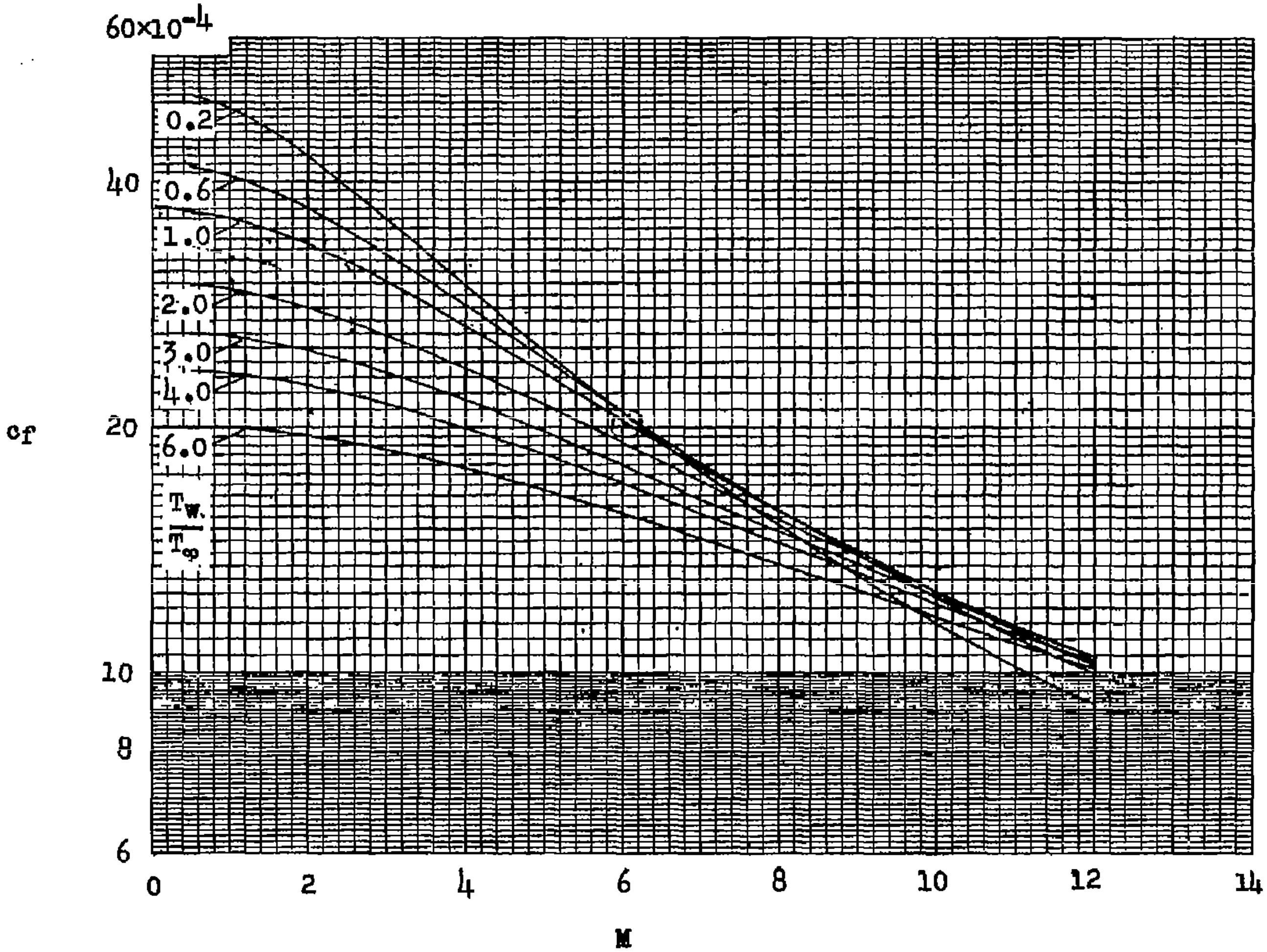
(a) Variation with Mach number for several temperature ratios.

Figure 2.- Values of Z as a function of Mach number and temperature ratio.



(b) Variation with temperature ratio for several Mach numbers.

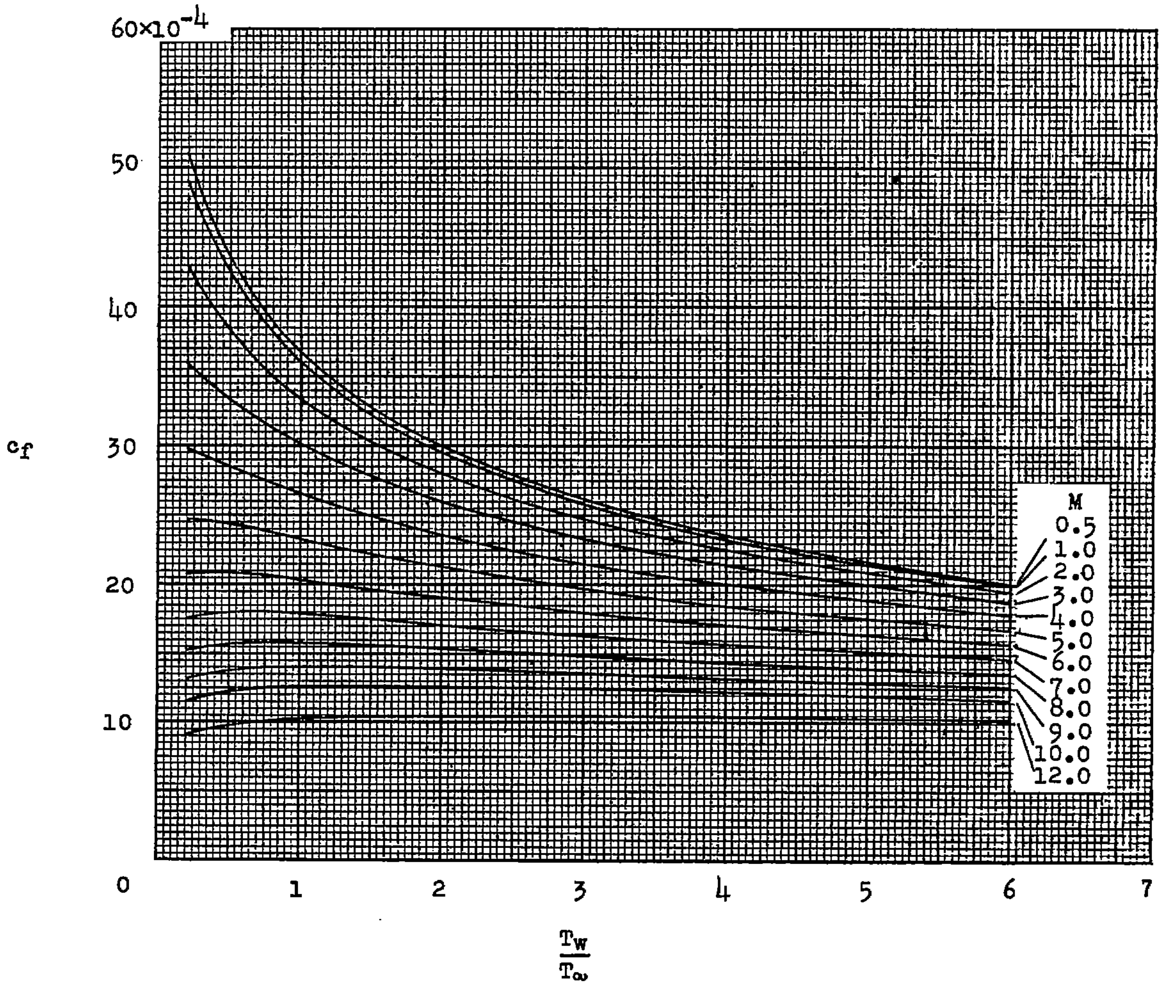
Figure 2.- Concluded.



20×10^-4
.0001 × 20
.0020

(a) Variation with Mach number for several temperature ratios.

Figure 3.- Local skin-friction coefficient for a Reynolds number of 1×10^6 as a function of Mach number and temperature ratio.



(b) Variation with temperature ratio for several Mach numbers.

Figure 3.- Concluded.

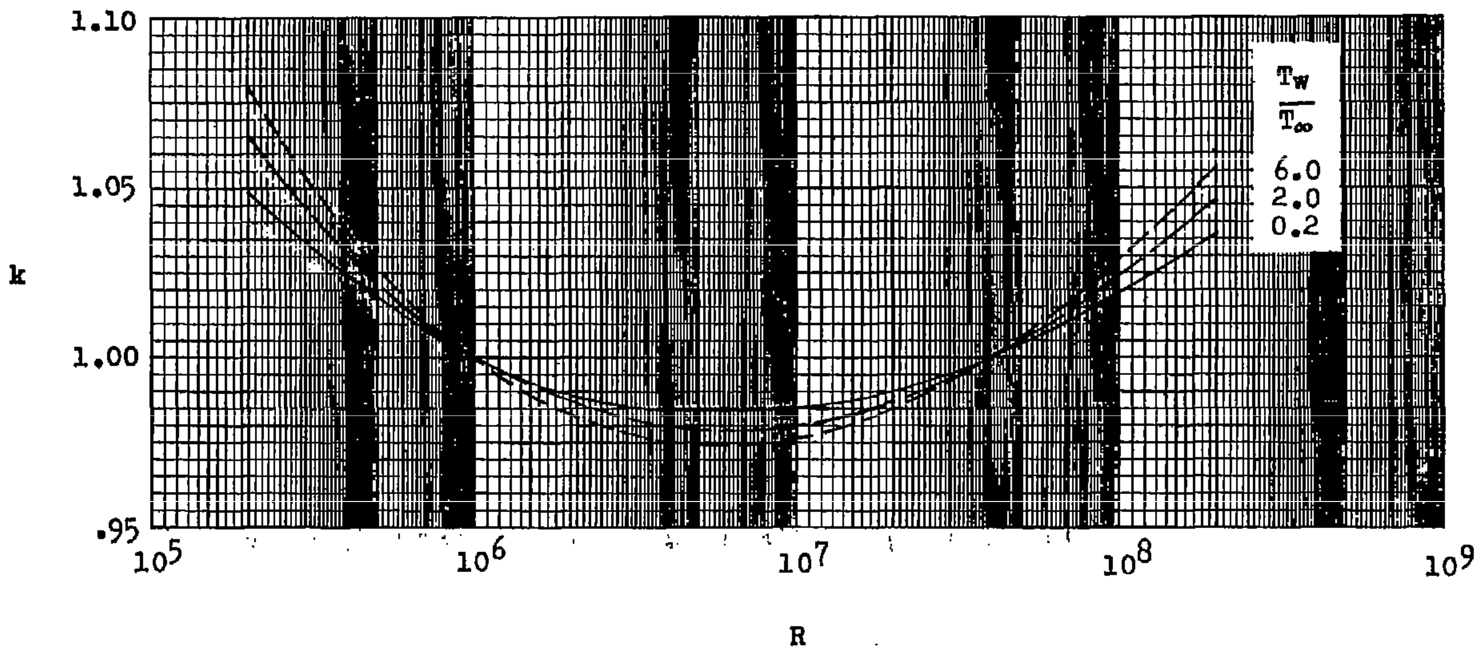


Figure 4.- Correction factor for all Mach numbers.

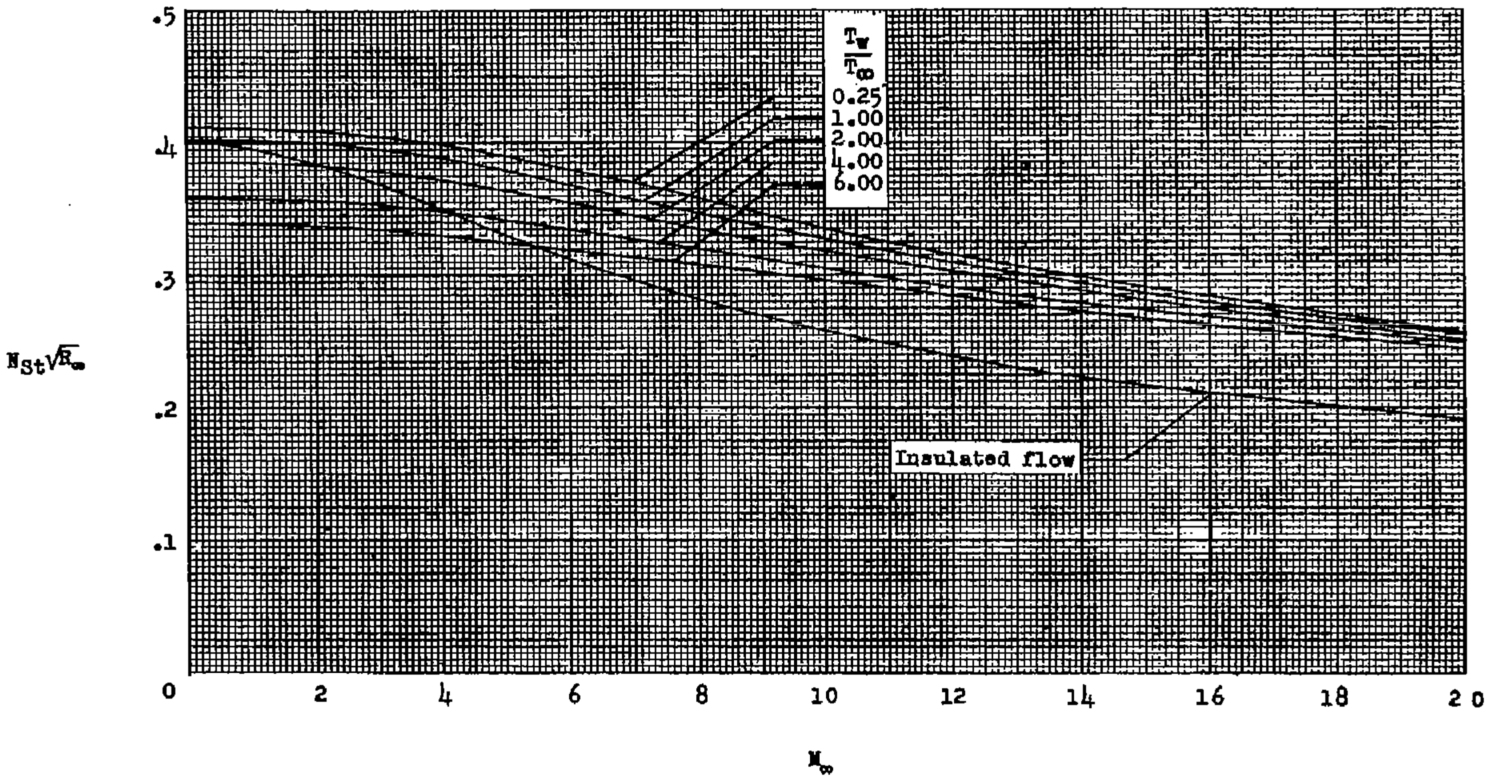


Figure 5.- Local heat-transfer coefficient for laminar boundary layer of a compressible fluid flowing along a flat plate.